Program to encourage critical thinking in children

Research carried out in schooling year 2013-2014

by

Udavi School in collaboration with Dr. Sanjeev Ranganathan
Introduction

About Udavi

Udavi school is an outreach school of Auroville that caters to village children primarily from Edayanchavadi village.

Udavi School is committed to provide and integral education as per Mother’s and Sri Aurobindo’s vision. The vision:
“There will be a primary class problem for the whole population... for Auroville. And that will be an interesting problem: how can we prepare the children, children taken from anywhere, who have no way of learning at home, whose parents are ignorant, who have no possibility of having any means to learn, nothing, nothing, nothing but the raw material, like that – how can we teach them to live? That will be an interesting problem.” – the Mother

Research Objective
How do you encourage critical thinking in children in a rural school?

Critical thinking will be explored through three-pronged approach – through puzzles and games, classroom interventions and classes in hands on electronics.

Research Methodology

In this project critical thinking will be addressed through three-pronged approach – through puzzles and games, classroom intervention and classes in hands on electronics.

Intended Research Outputs

Children will be capable of abstracting, mathematization, observation and reasoning. Children will be happier and more aware of what they are doing and why they are studying mathematics.
Preface

At the beginning of the 2013-2014 schooling year Udavi school started collaborating with Dr. Sanjeev Ranganathan. Sanjeev has a PhD in electronics and is a practicing engineer in analog and RF design. He is also passionate about cultivating and critical thinking and self-awareness in children and has volunteered with Asha for Education for over 13 yrs interacting with over 60 NGOs working in the field of science and mathematics. He partnered with our teachers for the entire year on various aspects of our math (and science) instruction and established an electronics laboratory in the school.

We have completed a successful year of many interesting experiments and felt that though we recognize him very much a part of our team, we also recognize that he continues to be impartial in his assessment of what we accomplished through our attempts. We have volunteered him for the responsibility of preparing the report as the author. Below is his brief personal experience that summarizes various aspects of this research:

I spent a month with the Xth graders in the June being part of their mathematics classes observing and towards the end of each class linking different concepts they learnt, e.g. similar triangles with trigonometry and coordinate geometry, geometry with algebra. I realized that many children struggled with abstract concepts they were introduced in 6th and 7th grade – fractions, algebra, word problems and a feel for what the result should look like.

I set up an electronics laboratory at Udavi to provide children an ‘exciting’ context to learn mathematics and as an exposure to a different discipline. I maintained connection with the Xth grade through a slot allocated for an electronics class, but focussed on addressing the source of their discomfort and worked with 6th and 7th grades in mathematics. I was fortunate to find teachers who not only let me take over the classes, but also support the initiative and fill gaps that I left.

I addressed fractions and number systems with use of hands on tools and games like pizza party and Denzel blocks. Together with the children we invented many games beyond the ones prescribed with the tools. We even looked at many abstract ideas like \( p^2-1 = (p+1)(p-1) \) and how to visualize these through blocks.

Strategy games that are short yet intense like those made by the European company Gigamic were a huge hit and so were disentanglement puzzles. The 6th graders put up a stall at the school fair with these games to provide a challenge for children, teachers and visitors a challenge to play a game with them. The interest in development of strategy was high enough that each child spent over 2 hrs holding their stall before taking turns to go and participate in the fair. They even developed an idea of
rewarding younger children who played well, even if they could not better them in the contest.

Children spent a lot more time with questions without being overly concerned with the right answers. Children spent time articulating questions and going between abstract and concrete. To take an example we talked about multiplication story (of 3x4: 3 bags with each 4 apples) and the fact that there are two corresponding division stories (12 apples in 3 bags and 12 apples in some bags such that each bag has 4 apples). We noticed how the first story came easily while we struggled with the other and learnt how to notice the quantities being asked and those missing to understand what kind of story was being told. This skill is fundamental to understanding algebra. We created a process of peer learning and children listened to each other. We started with simple exercises in repeating at random what the other said and later to processing what was said and converting one kind of story into another (multiplication into its division, etc). These listening exercises had an incredible impact on classroom participation, retention and being able to build on concepts of complexity.

This also led us to scientific phenomenon like speed, distance and time that have such a relationship. We also looked at density beyond a keyword and looked how we can make abstract ideas like mass and volume into more concrete seeing (volume) and lifting (weight).

I encouraged children to look at examinations as an opportunity to understand and grow. They were allowed to bring one sheet with any information they felt needed to be learnt by rote, allowing for exploration of understanding and application in the examinations. I also placed a score on meta-cognition of knowing if they got something right or wrong (and you would get a score if you were sure you got something wrong and it was). They even wrote the examinations again to see if they had a better understanding after a few weeks.

We spent a fair bit of time talking about being able think of whether a result makes sense and catching common errors. We worked on EBD (Education By Design) and the 7th graders presentations on fractions, decimals, algebra, etc to highlight where they commonly made errors. The process of EBD of creating quality criteria before starting the project and using a new process (increase/decrease/retain) of feedback made inputs less personal and more constructive.

Being able to talk about algebraic expressions as stories and write stories into algebraic expressions was good, but it took a whole new perspective when we worked with geogebra that linked geometry with algebra.

In the first term I also worked on English stories the children wrote to see if they were logical or were flowing from one context to the next.

The 10th graders made many small circuits on bread boards including LED oscillators, 555 timer circuits (which they also used for make musical
notes), seven segment displays, counters, taking things apart and trying to repair them and programming micro-controllers. While in action I also received support from AIAT and IITM who leant us oscilloscopes to help ‘see’ electronic waveforms and cultivate intuition of what AC/DC signals look like.

The electronics lab was also utilized by some children from Deepanam school and a couple of children from TLC. There were also some classes when we were able to have an interaction with these children and the 6th graders at Udavi. The innovations in mathematics and classroom instruction were complemented with similar experiments in Isai Ambalam school.

Teaching at Udavi school has been a grown me as a teacher and I hope to continue working towards progressive and integral education here.
Acknowledgements

This research stands on the shoulders of work done by many individuals engaged in joyful learning with children. I has been moulded by my experience of interacting with individuals (and their organizations) working on joyful and engaged learning over the years of volunteering with Asha. I would like to acknowledge some of them, Jane Sahi – for introducing me to the poise and self-inquiry of an educator, Ravi Aluganti, Arvind Gupta - for introducing me to tap into creativity of children for their learning, Anu and Krishna – for introducing me to integrate mental work with hands on work, Nandlal and Mahesh who helped me see the many invisible children who do not benefit from the current mode of education.

Many writers have also impacted my world view and I would like to acknowledge some of them- Tetsuko Kuroyanagi (Totto Chan) for a vision of a different school, John Holt - for helping me question an education system though I was successful at it, Balaji Sampath - for introducing me inquiry based teaching, Adele Faber & Elaine Mazlish for helping me listen to children, Danniel Quinn and Fukuoka who made me wonder what we are doing as a society.

I would like to acknowledge Lata and Prashant who used their knowledge and identified and introduced me to Sanjeev at Udavi. I would like to acknowledge Sanjeev for his faith in me and being extremely open to some of the wild ideas executed in this work.

I would like to acknowledge the children at Thulir for their engagement and providing me the only practical teaching experience I had when beginning this work.

This work would not have been the same if I had not undergone “Stewardship for a new emergence” workshop even as I was beginning this work. I would like to acknowledge Monica Sharma who helped me tie inner wisdom with my work and address systems that we take for granted. I would also like to acknowledge her for introducing me to many distinctions that helped me not get stuck with outputs and look at outcomes and impact. I would also like to acknowledge her for helping me see how I should frame my classroom – as a safe yet engaging space for children.
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1. Games

1) Games in classroom

Generally, only a few other math games have some penetration in the lower grades in most schools. In the higher grades the only strategy game is chess, but it caters to a small number of children due to its sharp learning curve and long playing times.

I used disentanglement puzzles and strategy games as an ice-breaker with the children. I found time at the beginning of the year when classes had not kicked off with full steam I was allocated time from the French class to interact with children from 7th to 10th grades. My assumption then was that I would work with children of these older grades.

The disentanglement puzzles can be solved once, games though could be played over and over. It is important to pick games that can be learnt quickly, played over a short span of time and yet require reasonable strategy. I introduced award winning strategy games like Quarto, Pylos, Gobblet, Cathedral, Batik that the children had not seen before and fit the constraints mentioned above these worked to charming effect in getting the children engaged. It also did a lot more set precedence for the classes to be engaging and fun. This meant that when it wasn’t we had to innovate. The idea of creating a game has really appealed to the children and we did some experiments with innovating with learning material e.g. pizza party to create games. This will be illustrated in detail under the fractions section.

With the 6th graders we used games once in a while in class as a refresher after we had been working on something intense for a few days. For the first six months the 9th graders asked for a new game every few weeks that they would swap out for an older game and play over their breaks. I also used up most of my disentanglement puzzles to a similar scheme.

2) Putting a games stall

As part of my effort to create an engaging atmosphere for kids to think I had introduced many board games requiring strategy to the 6th grade at Udavi. With Christmas was the annual fair at Udavi and the kids put stalls with games involving luck and concentration. I inquired to the sixth grades if there were any kids who wanted to put up a stall for the board games and puzzles. I thought it would be an interesting exercise to increase the mental stamina of the kids to be able to play with different people throughout the morning (9:30 a.m. to 12:00 p.m.). The teacher had also informed me that the kids tend to abandon the stalls in order to try out other games so I had to have backups (and play myself if all else failed :)). Seven kids volunteered and even spelled out the games that they like and
would like to host. Four children volunteered for the games stall and three for the puzzles stall.

We had a practice session in which we polished the puzzles while solving it and renewing our understanding of the games and the strategy of each game. Though polishing the rust off games is an effort, the kids generally had a good time and talked about the posters they were going to make.

I got busy with trying to make a game to make fan fixture to get a ballon go through hoops...so come game day the kids had made posters about their games, one had dropped off and one had joined. The booths had visitors almost all the time. The kids kept up to their booths. None of the girls in the puzzles counter budged the whole time, older kids, alumni, those from other schools tried their tricks, but the girls held firm - kept to the time allocated for a puzzle independent of taunts, pleads, muscle men who tried to pull things apart. They also and kept rotating the puzzles to avoid the passerby who may have been waiting to see the solutions.

The kids had been practicing the games and had mastered it by then. They played some practice rounds with the 10th graders and one of the kids quietly told me in a surprised tone that the 10th graders are actually smart. One kid who had not volunteered initially saw the fun and joined in. They kept to it till around 11:30 when a couple of them really wanted to try the other games. I relieved the kids in one set of games and the 10th graders followed to play with me. I then handed over the stall to the 10th graders.

The kids had interesting feedback about their experience. One of them said that he only got beat twice, but he gave tokens to kids who had played well especially the younger ones. One kid admitted that he was beaten when he was distracted. Another mentioned that one kid from the other school I teach was good at puzzles and tried to help other kids out. One of them had mentioned that they didn't play any of the games, it was ok, but they didn't get dairy milk chocolate. One of the kids said that he learnt that he made up good strategy and was difficult to beat. Still another talked about the game that didn't get used and said he wanted to use that. One child recalled an experience in which an older child called him a cheat after he won the game. They had a rematch and he won again and the person still called him a cheat, he let her play again and this time when he won the person quietly left. One child said that she will try to be more relaxed and have more fun, another said that she would be more serious.

When school reopened today I took 20 chocolates to class. I told the ones who had maintained the stalls that since they missed out that day they can have the packet and decide how to disburse the same. It would be nice if everyone got something, but even that was up to them. I only wanted to know the fraction of the total each of the 15 children finally got. The kids decided to give a chocolate to everyone and then realized that there were more. They asked me how to disburse the rest. I told them it was up to them. One of them suggested that they keep it, another pointed out that there were only 5 chocolates and 8 of them. The next suggestion was that there were 4 pieces in each chocolate and we should split it evenly, another intervened and said that this would not work. Still another suggested that this was a fraction problem and each dairy milk needed to
be split into 3 parts for each person to which a child pointed out that the chocolate had only 4 pieces. At this point I walked out of class with a smile. For all the puzzles and problems we make up, nothing beats real life!
2. Classroom Atmosphere

Great changes in settings outside of classroom need to be complemented with changes in core academic classrooms to create an atmosphere conducive to open up children’s learning. Together they create the greatest potential of an impact on children.

The NCF (National Curricular Framework) 2005 addresses mathematics in the following way: Mathematics - Developing children’s abilities for mathematisation is the main goal of mathematics education. The narrow aim of school mathematics is to develop ‘useful’ capabilities... The higher aim is to develop the child’s resources to think and reason mathematically, to pursue assumptions to their natural conclusion to handle abstraction. It includes a way of doing things, and the ability and attitude to formulate and solve problems.

This work is looking at ways of realization of this vision and is looking at ways to address the higher goals of abstraction and understanding while keeping in place competence by providing rigor (practice with understanding in different contexts). The classroom interventions and innovations were focussed on mathematics classes.

a) Present situation in higher grade (7th-10th) classrooms in Mathematics

On observation of the Xth grade classroom I found that the goal of the teachers was to help the children score the most marks in state board examinations. The questions in the state board examinations use the problems from the exercises in the textbook verbatim. With this in mind, the teachers have found the most efficient way to address the examination is to extract relevant formulae from each chapter and solve every problem of the exercises in the book on the board. The children copy all these solutions in their notebooks neatly.

The teachers are helpful and after solving the problem on the board go over each step (which generally involves some algebra) and repeat how the numbers in one equation changes in the next. The children who are able to follow the steps have a guide to try to do check that this is what they would do at each step. The children who are unable to follow the steps learn as many problems by rote and hope that the solutions they have memorized come in the examination. This approach also allows the teachers to complete the syllabus at their pace.

While the approach attempts to addresses the narrow goal in the hope that children will get a good score in an examination that tests capability, the higher goal is unaddressed. It is, however, not obvious if year to year this approach has proved its utility to help all (or most) children in class secure a good mark in the government examination.

There are of course limitations in the approach of being unable to engage children and a need for teachers to focus on discipline and copying things accurately in the notebook. The children also learn each concept in isolation and are unable to connect concepts even within Mathematics e.g.
after doing solving for slopes of a line as \(\frac{y_2-y_1}{x_2-x_1}\) for a couple of days, when introduced a property a line to hold ratios of two corresponding sides in two right angled triangles (big or small) drawn with their hypotenuse on the line, they found the property magical and could not see that it was slope.

Till 6th grade the approach followed by the teachers at Udavi is targeted towards joyful learning and all round development of the child without examinations. To ensure that examinations do not come as a shock to children when they get to Xth grade this system is followed from the 7th grade onwards.

b) Right answer to right approach
In almost all grades in a Math class the focus is on the right answer. No sooner has a question been posted a child will pop out the answer and enquire if it is the ‘right answer’. This makes it impossible to have a conversation about a problem, let alone analyse what can be learnt from it.

The first step was to change the conversation. I consistently stated that I didn’t care about the answer, what I cared about how to go about it so it helps us learn not only to solve this question, but also others like it. This took children by quite a surprise initially, but later became a norm in class with children themselves commenting that Anna doesn’t care about the answer, only about how you did it. The advantage with the approach was the fear or making mistakes slowly started going down and this opened the possibility to try new things.

For the solutions children offered I asked them to create a situation where what they wrote would make sense. It started with simple subtraction sums and which helped me move from sums to word problems. Many children had issues tackling word problems as they need to work out for themselves what they are expected to do.

I focussed on multiplication and division that the children at the 6th and 7th grade take for granted. I picked these because they form the basis of many concepts in science including distance =speed*time mass =volume*density even when we move to more advanced concepts in electronics like voltage= current*resistance Most children fumble when two quantities other than the two quantities in the right hand side are given to them.

I wanted to bring forward the idea that a multiplication story can have two equivalent division stories where the quantities mean the same thing, but you are giving information about only two of the quantities.

Multiplication story: One basket has 3 mangoes. How many mangoes are there in 5 baskets? Its two equivalent division stories
Division story: There are 15 mangoes, if there are 5 baskets and we divide the mangoes equally, how many mangoes will be there in each basket? Division other story: There are 3 mangoes in each basket. How many such baskets will have 15 mangoes?
Almost all children in 6th grade had only heard and were able to produce only the first division story and are unable to produce the second division story.

Once children start getting a hang of the stories we can build it towards the science concepts and given a story like:
Story: A car covers 60 km in 2 hrs. What is its speed?
The children are able to understand what needs to be done and even create an alternative division story by using the solution:
Other Story: A car travels at 30 km/hr. How long does it take to cover 60 km.

The idea of focussing on creation of stories rather than just on the answer was one way to help children stay with a problem which helps work towards patient problem solving an essential quality to instil towards critical thinking.

On speaking to teachers of younger grades I realized that often teachers and students use only one story for subtraction e.g. 6-3
Story: I have 6 pebbles, if you remove 3 pebbles how many pebbles are left.
Other story: There are 6 pebbles, I take away some and there are 3 left. How many did I take away?

The equivalent addition and two subtraction stories would be:
Addition story: I had 3 pebbles, I received 3 pebbles from my friend. How many pebbles do I have now?
Subtraction 1: I had 3 pebbles, I received some from my friend. Now I have 6 pebbles. How many pebbles did I receive from my friend?
Subtraction 1: I had some pebbles, I received 3 pebbles from my friend. Now I have 6 pebbles. How many pebbles did I initially have?
The two stories of addition-subtraction and multiplication-division turned out to be a great stepping stone for Algebra. Allowing children to understand in context what they were solving.

Unlike an answer that is one number and a conversational killer. A story is a conversation starter, even if the story was not what was intended it still allows for processing either by the facilitator (teacher) or another student to keep learning from it.

Many children do not understanding division well and with the introduction of fractions this situation is compounded. Division stories helped to bring this issue forward and we could build on it leading to fractional answers (6th grade) and in using fractional quantities (7th grade).

Physical stories of speed, time, distance and density, volume, mass were introduced in time to make such stories interesting and relevant. It also helped to introduce the fact that math is the language of science and quantities in real life need to have units to make sense.

c) Improving **listening and peer learning**
If children develop their skill of being able to be concentrated in the moment their learning improves significantly. One was to improve this is for them to focus on something immediate like the spoken word and be able to process it mentally.
The first exercise in doing this was through the multiplication and division stories mentioned in the previous section. Instead of giving instructions and questions/stories I asked each child to come up with their own multiplication story (question) while the others would listen to his/her story. Once all the children completed their stories we would go back and ask children to repeat the stories of other children. We initially did it in reverse order i.e. the last person who told a story repeats the story of the first person, once the children are able to do it and notice the pattern. We started to randomize the story they needed to tell.

This was first done by splitting the class into two groups limiting the stories that a child needed to remember of others. The lack of attention of some of the children became obvious to them and they tried to look for approaches to improve it. I allowed children who were really struggling to write down numbers (but not the entire story) and the operation of the story (as they understood it).

It took a couple of classes before the children got a hang of it and then we were able to build on it by one child telling a story e.g. multiplication, a second child telling its corresponding reverse (division) story and a third child completing the story (second division story). In case there was something off other children could reprocess the information and act as correction systems. If any of them got confused they had to convince each other.

Once the process of listening was introduced we were able to use it in many different contexts e.g. to listen while other children to relate the experiments they liked at a science fair and to be able to remember each other's choice at the end of the sharing. Children also started to distinguish between wanting repetition because they didn't understand or they didn't listen. Of course, having a classroom free of fear is imperative for such a process to work as it allows children to be honest without fearing of repercussions on being honest.

d) Guided discovery
Guided discovery a process when children are part of the creation of an idea, concept or clarity energizes the classrooms and opens it up as learning of equals. It is described with an example from my notes about a class:

A day after I had worked with the sixth grades on density of objects I went over their guesses of what the volume of an object should be based on its mass (in g) we measured with a weighing scale and whether it floats or sinks.

Given that they claimed to know density and knew the density of water was 1 g/ml. I wanted to see if the kids could guess if the number for the volume (in ml) would be greater or less than the mass that we measured in the class (in gm) based on if it floated or rank.

Most sat on the fence and put the same number for mass and volume. At this point a child shared her insight - Anna, mass and volume is the same.
I froze for a couple of seconds and then I inquired if there were others who could help answer her question.

A few hands went up and the children said mass is measured in kg or g and gave examples of potatoes & tomatoes and volume is measured in ml/L water, oil, diesel, petrol. I wondered if the problem was that the unit of volume is always used with liquids. I inquired if they thought solids have volume and liquids have mass...this question made them uncomfortable. Their thinking was liquids were measured in L so if another thing that is not measured is needed, you can take this number, similarly solids have mass ('weight') that is measured in kg/g and since that is all that is measured (reinforced by my 'clever' experiment) the other quantity is what we got from measurement.

There was also a question of why we bothered with the measuring scale. I gave her the battery and eraser that we measured before (one in each hand) and asked her to tell me how many erasers would be needed to weigh the same as the battery. She said 2 to 3. I asked her to look up their weights. The battery was 18g and eraser 3g. She said she understood that we do it to compare accurately.

Another child pointed out that the battery was also much bigger than the eraser and so naturally its weight was more...

We then did an exercise I asked children to close their eyes and placed different objects in their open palms (in two cases I was also supporting some of their weight) and asked them to guess how big the object was based on its weight, we used pencils, magnets and a large cardboard box. I had to pause when one child lifted his desk for the volunteer to weigh. All the estimates were significantly off (and none of the children cheated).

At some point a child came up with the following distinction - 'what I can feel is mass and what I can see is volume'. This seemed an aha moment for everyone and there were smiles all around.

I also clarified that it might be confusing that we seemed to have been comparing two different quantities ml with g. I reminded them that we were doing was finding the density g/ml and then comparing it to 1 g/ml which was the density of water. If the calculated fraction was a proper fraction it would float and if it was an improper fraction it would sink.

I asked if they could think of an object whose volume can increase significantly, but the mass changes only by a bit or even fall. It needed the hint of a ‘party’ to get to a balloon. We then talked about how to distinguish a balloon that is filled by a helium cylinder and that filled with their mouths.

In the next class, I asked them if they remembered the difference between mass and volume. The children said ‘mass is weight and volume is space’. But, what was more interesting is how they said it mass is weight (both hands down palms facing upwards), volume is space (both hands moving away from the body).
I shared the distinction of the child, 'what I lift and feel is mass and what I see is volume'; with the 5th graders in another school and it seemed to clarify the trouble they were having too.

e) **Tapping into creativity**
I introduced games and puzzles as part of the classroom. However, in solving a puzzle or playing a game a child is still the user of someone else’s creativity. Mastery comes in being able to create the same.

a) *Creativity: Making their own games*

Here is one of the first things we tried this on:

**Pizza Party**

Most children in my class know what a pizza is (not all like it). The kids that don't like pizzas assume that its a dosa/cake. The game pieces are reasonable for the price, but the suggested games need work.

*What it has:*

- Base cards of fractions 1/2, 1/3, 1/4, 1/6, 1/8.
- Pizzas divided into these fractions e.g. 4x1/4, 6x1/6.
- A die with these fractions.

One often wonders if doing these fraction games is really worth the time and I started a conversation with the kids of what we learnt (not what we did) from the activity. With feedback starting from 4 pieces of 1/4 makes a whole. This helps reiterate 1/4 means you cut the pizza in 4 pieces and take one. Similar ideas continue for 2 pieces of 1/2 and 8 pieces of 1/8. Then we move on to expressing one set of pieces in terms of another, 1/4+1/4=1/2, 3x1/6=1/3 and my personal favorite 1/2+1/3+1/6 is a whole pizza. Wow, made my day.

I invented a few of games to demonstrate that we didn’t need to stick to instruction and create our own. Here is one: selecting pieces to make a pizza in turns, but you pick the piece for the next person to play. The one who completes a pizza gets a point.

The children then created the following game:

One person rolls the die and picks the piece. Then the next person rolls and gets a fraction and picks up a piece which is the difference of the two pieces. If no such piece exists then he or she picks up the largest piece less than the result and the smallest piece larger than the result. If anyone finds a more appropriate piece they get the point.

b) *Creativity: Education by Design (EBD)*

The children are keen on doing something beyond classroom. I had introduced electronics as such an outlet and they had taken to it with enthusiasm bringing what they had motors/lemons/led display lights.

I was looking for something that they would feel is more directly academic. I found the computer lab was available for two days in the week and I decided to adopt a project based approach also called education by design (EBD).

The goal was for each student group to make presentations or charts (the discussion started with some wanting to do it on paper, but eventually settled with doing it with libreoffice).

I was hoping that the children would get a chance to think of their work, quality of work and perhaps even learn to catch their own mistakes.
The topic was, “To identify what kind of errors can be/are made in different areas”. The areas I offered were fractions, decimals and algebra. Each child chose the area he/she wanted to work on. They then split themselves into groups based on who were aligned with their areas. There were finally 6 groups with 2-4 children each.

**EBD: Quality Guideline**
Before the work started we agreed upon a quality guideline on what they like to see in the work. The ones they came up with:
1) Neat, clean, nice
2) Should be free of spelling mistakes
3) Should be free of errors (unless they are used as example)
4) The idea of what is being presented should come across clearly
5) People presenting should be able to explain what they have done
6) Creative
7) Interesting

The topic itself was quite difficult and I wanted to see the interpretation of the topic by the children.

**EBD: Execution**
In the phase of implementation kids really struggled with the idea that a presentation could be about mistakes. They understood making examples of how things are done 'right', but what they were supposed to do with something that was 'wrong'...

Many made presentations that were Q & A or multiple choice questions, some even copied examples from the book (ah, but they finally read the book on their own :), possibly assuming that since I didn't teach from the book I had not looked at it. It was still more amusing when there more conscious partners announced their partners actions. I just pointed out that they would be toast when it came to creativity guideline :).

I also realized that unlike electronics where you need more block time to build something and get it done time allocated for the project, but planning time away from the computer was more useful than a full block of 1-1/2 hrs as children tended to drift off from topic and concentrate on trying templates and beautification.

Consequently, after the second class I started using half the time of the class for other instruction. They worked for 5 classes (2 full, 3 half) on the project when seemed like they were saturating. I asked if they needed more time, but the kids felt that they were done.

There was one group that spent most of their time away from the computer trying to figure out how to solve algebraic puzzles. They were convinced that once they got it they will be able to write it up within a class (they took 2 half classes).

The longest presentation was on decimals where they covered examples of multiplication, division, addition subtraction and word problems. They did cover an example of when they thought long division goes wrong. For good measure, they even threw in a touching thanks where they thanked Sam/Pai for teaching them decimals and me for inspiring them.
The good part was that most of the kids were engaged in the activity in some capacity. There were a few kids who were not engaged and a couple of instances when I needed to step in. I realized that I had assumed that since the children picked their groups they would be comfortable with each other and decided to focus on ground rules for working with each other (not calling each other names).

**EBD: Talking the walk (presentations)**

It was time for presentations. In each group there was one person who had pulled more than their weight and would have dominated the presentation. It was time to weed them out :).

I asked each group to volunteer one person. Not surprisingly the children who the group expected to present volunteered (and in some cases were volunteered). I made them all judges for the presentation. The reaction of the kids when they found out that the judges were not presenting was amusing. As we walked over to the computer center, one girl even commented that she thought that I had asked a volunteer to make a presentation for the group. Cute.

While many presentations were limited in content, most were original and demonstrated some thought process. Most of the children were engaged when they were going over the presentations and asked a lot of questions (especially when it needed corrections). Interestingly, the kids were able to identify their error (goal of the project) and occasionally think on their feet and even suggest how it could be corrected.

By and large the discussions were centered around how mistakes creep in, even if they were unintended at times :).

During the creation of the presentation one of the children had been called a daydreamer by her team and I had stepped in and worked with the kid. It was interesting that this child stepped up and made the presentation for her team.

There was one amusing instance in which a child had interpreted an algebraic puzzle of dogs and cats eating 58 biscuits with 5x6+4x7 even though it was unrelated to the question and instead asked, well isn't it 58 to which another child replied well so is 4x10+3x6, why didn't you put that instead. It was interesting that they were talking about two points on the same line without realizing it. But, that's a geogebra story for later.

It took us two half classes and 1 full class to complete all the presentations. I kinda rushed in the end as they were loosing interest.

**EBD: Giving Feedback**

Feedback with kids to kids can be a very tricky affair it puts a child in an apparent position of power and not one that most kids can handle. However, since I had made 6 judges it was time to give feedback.

I had done a series of workshops with Monica Sharma (stewardship for a new emergence) and felt that there were many aspects of my teaching and class that originated from it. It was time to put one more to test - giving feedback.
Each feedback needed to start with
1) I learnt from your presentation....
2) One thing that could have been added to the presentation...
3) Go through the quality criteria list and state
   - increase
   - decrease
   - retain
for each of the 7 items on the list.

The first feedback came from a cocky kid:
"I learnt from your presentation how not to make mistakes."
I asked him to explicitly use I for what he should do, it became
"I learn from your presentation that I should not make mistakes", his
hesitation was obvious, but one look at me and he knew that both he and I
knew his work. The sneer turned to seriousness.
The remainder of the feedback was efficient and to the point. It also made
feedback specific and multi-dimensional instead of a like or dislike.

Of course, the children were quick to note that 'retain' was 'good'. I
reminded them that all mattered is if they knew how to improve their
presentations, but I don't think this point landed :).

I added - "May I give you feedback?" as a first step as there can be
relationships beyond my knowledge that made it impossible for some kids
to take feedback from others (with a limit of 2 no).

Few interesting notes:
1) Judges who had not paid attention to the presentation were less keen
   on trying to wing it.
2) The judges found what I learnt from your presentation tough and were
   thrilled when they didn't have to give feedback. Groups enjoyed saying
   'yes' to feedback.
3) It made the feedback efficient without having to go into every mistake.
   One judge had written 2-pages of errors she had found with one group
   which she didn't get to use.
4) Judges were able to give feedback to their own team which was a very
   big step for children to be able to do.

c) Creativity: Number Line
Creativity can also be introduced in every activity as an example we were
working with integers (positive and negative numbers) on the number line
and realized that we had many hops back and forth to go over a series of
positive and negative numbers. We made pictures for the same on the
number line. Could we just use the picture and arrows on the hops to
predict what the numbers were? The children each created a challenge
based on the numbers they had and offered it to other children. This also
brought up the question - when could we not predict the numbers (when
consecutive hops were of the same kind)?
3. Puzzles

a) School puzzles
After my work with the Xth graders in the first term I focussed on getting the basics right with 6th and 7th graders. However, I had put a process of putting up puzzles that could be related to the curriculum on the school notice board. Once the children were hooked on to it I was able to continue to get them to think about something different each week even though I had no classes with them. I put up around 40 puzzles over the course of time and got 50 responses to 20 of them.

Two things were of interest to me, one, that we started having a conversation about a puzzle without putting pen to paper, this could be regarding the context, regarding guesses they made that we could logically knock off, etc. Second, the puzzles attempted slowly moved from only those related to visual puzzles to those that were related to algebra. Though only towards the end they were able to get the abstraction of algebraic puzzles into an appropriate equation it was visible growth for these children.

A collection of these puzzles are available online at: https://sites.google.com/site/puzzlesatschool/
They now only include the puzzle, but different ways to look at it and solve it which can be of interest to 6th grade onwards.

b) Quiet time
I had been putting up mathematical puzzles at Udavi for about 3 months in the building area of sections 7th to 10th grade. I had been getting responses mainly from a few kids in the 10th grade. The other kids have not been as involved in solving puzzles. I noticed that in the younger grades (6th) the teacher does put up puzzles occasionally and thought it would be interesting to pursue solving puzzles there.

I created a couple of puzzles that were visual (match stick puzzles) and put them up in the 6th grade. Again I found that 2-3 children were keen on solving it, but the rest were not able to. We then had a class to solve the puzzles.

I had been working with these kids for a couple of months now and had noticed that the classroom gets pretty noisy as the kids who 'got it' were too eager to sprout out the answer and the kids who had 'not yet got it' were all to comfortable not having to think. I had been working with the kids to do individual work in the notebooks without having to raise their hands or talk allowing me and the other teacher to go around and look at their work. The classroom had been marginally quieter at times. Of course there is an equal mix of group activities (games) or discussion time (sometimes moderated) and hearing about each others work.

In this class, we took a step further. There were enough puzzles for someone to continue solving if they finished one and they were asked to work on this and try to create a quiet space to allow them to think. The teacher and I went around to see how the kids were doing. Occasionally, we encouraged the class that they could do it and just needed to relax and
find their quiet space.

That class was magical, the children were able to find their space and over 80% of them were able to solve the puzzles on their own. The other 20% needed individual time from us to ask them questions that led them to think of the answer and eliminate the impossible that they were stuck with.

Unfortunately, with everything magical, it has been difficult to replicate. But, there are now more children that attempt puzzles and also ask for it when I am in class.

We have now been slowly moving from visual puzzles to ones with numbers and in a recent class the seventh grades were thrilled to discover that though they could not solve algebraic puzzles, they could create them and each child created 5 or more puzzles. I used these puzzles as assignments for children in other schools who had come to the point of solving algebraic puzzles.
4. Examinations

a) Altering expectations and minimizing rote

Examinations are a wonderful opportunity to innovate and emphasize what is important for the children. Assessment can be continuously done in class through formative assessment and in an interactive class where the children are learning through Q & A it is easy for both the teacher and the student to track a child’s mastery in an area.

Summative assessment or term examinations can then serve other purposes. The 7th graders at Udavi had experience of the first term examinations and had developed a fear of it.

Having altered the classes towards understanding and application the examinations also needed to be oriented towards the same. There were very few ‘sums’ as the children had come to expect and they needed to interpret the questions to understand the kind of concepts they would need to apply.

The examination was clearly at a much higher level than they were used to and four things were done to comfort the children and help them through the process:

First, the children were given access to read the paper and clarify anything, including the English that they could not understand. Since some children have a difficulty with language it allowed them an additional time to get familiar with the questions. They were not allowed, however, to write or copy the examinations. Though, of course a few children, tried to remember a few things including the puzzle in the paper.

Second, was to allow the children a cheat sheet. A notebook size paper with one side filled out with any information they felt would be useful to them. This sheet could have been used to consolidate their learning, put in any information they would need to remember or memorize, essentially anything they needed by rote formulae, tables, example problems, anything that they would require. An extreme of open book examinations in some courses in colleges requires people to know how to process large information and find what is appropriate. This felt too much for children to do in an examination and a single cheat sheet seemed appropriate.

Third, the examination paper was made for 120 marks, but the children would be graded only out of a 100 independent of how many questions the attempted.

Fourth, were additional points for meta-cognition as described in the next section.

It was interesting that every question was solved by one child or the other, their combined wisdom was impressive. The results of this examination didn’t seem much different from the results that were obtained in regular examinations with scores ranging from the 20s to the 90s, with a mean of around 60. Around 30% of the children could not figure out how to consolidate the learning in class to be useful in an examination and in the last minute copied what an enterprising child in their subgroup had done. Even the enterprising children didn’t use their cheat sheets as they had decided to copy formulae from the textbook and not could not move from information to application.
b) Moving towards meta-cognition

One of the bigger issues teachers face is the lack of meta-cognition in the children. It is difficult for a child to be rigorous if he or she believes that they already have mastery over a topic. Unfortunately, this is also an aspect that teachers don’t address or measure.

What we measure is what we move, so I decided to place a value on self-assessment.

I asked the children to grade their work and write down ‘Yes’, 'No' or 'Maybe' indicating their confidence that they had done it right, wrong or didn't know on their question papers. They would get an additional 2 marks for every correct assessment of their attempts with ‘Yes' and 'No' and lose 2 marks for every incorrect assessment. They could play it safe with 'Maybe' and neither lose or gain marks.

This exercise was really good to understand attitude of children to their work. It also brought up some interesting introspection from the children regarding how they perceive their own work.

I found that the children who already do well in class also have better meta-cognition. The children with learning difficulties were also aware of their limitations. But, majority of the class either refused to take a risk or could not get additional net marks as they got an equal number of assessments right and wrong.

This exercise also started conversations regarding how a child can know what he or she has done is correct or incorrect. How do we stay with a problem and find ways to cross check our answers rather than rushing to the next problem?

We were also able to build further on the area of meta-cognition as we approached the end of the year.

c) Using examination as a context for rigor

A very common word children use is ‘finished’. It is not an idea that something has been given thought and completed to the best of the ability. It is an idea that something has been attempted and effort on it can cease. Even if more time were available the children will not go back and look at something. This is particularly an issue with math when sums are finished and often nothing is learnt in the process. Patient problem solving which is the need of society requires that this attitude be addressed.

Self-assessment had already made the children spend a little more time than they usually did on problems, but following the examinations I talked about each question and how to approach it (and eventually even solved it on the board).

I place no emphasis on copying everything on the board to the notebook. I mention it is important to write what they understand, but processing and documenting is an acquired skill and much as I would have liked time to go over each problem I realized that I had lost the crowd and was faced with the ‘finished’ attitude. As far as most children were concerned the examinations was over and I was beating a dead horse.
I opened an opportunity for children to do better than they had done before and gave an option to rewrite the examination. I told them that if they did better than they had done before without the cheat sheet I would average or replace their earlier score. Naturally, the questions were now examined more closely, discussions on how to attempt the questions started and now the children wanted to check who had taken notes (none had).

Only 8/17 children were able to utilize the opportunity to do better than they had done, but everyone had tried to remember what the questions were and reviewed the areas that had been covered in the examinations. Without the meta-cognition scores the children who do extremely poorly and knew it did not benefit. One child had started working serious and bettered her score significantly and get the highest in class, this seemed to have boosted her immensely and they became more engaged and rigorous than ever before.

d) Make your own examination

I had met a teacher who corrected 10th grade board papers and said that he would fill out answers if a child was failing in the examination as failure unnecessarily ruins the child's life and confidence. He had suggested that children should just be asked to write whatever they know. I had started preparing graded papers for different children in the final examination...but I wondered if I holding back from experimenting the whole way.

For the term examinations there was scope of formative assessment being useful for areas that I could work with the kids on, but this is not the case with the year end examinations. Perhaps, like a Dojo it was time for the kids to show what they could do.

I gave the areas we had covered over the year and a score for the questions in that area. For the Udavi 7th graders this was:
A=(addition), S=(subtraction), M=(multiplication) and D(division)
Integers - ASMD- 10 marks
Fractions - MD - 20 marks
Fractions - AS (including using %) - 10 marks
Decimals - ASMD - 10 marks
Algebra - Curves, Puzzles, equations, story for equation - 10 marks
Geometry - rotation, angles of a triangle, relational (complementary, etc) angles - 10 marks
Practical Geometry - 10 marks
Ratio/Proportion - 10 marks
Area - 10 marks
Puzzles - 10 marks
Anything else they could think of could be for another 10 marks.

The guideline for making a question in any area was that a 'sum' is for 2 marks,
a word problem is for 4/5 marks,  
a picture question is for 2 marks,  
a picture question that involves algebra is for 4/5 marks and  
the puzzle can be just one and is 10 marks.  
To cover 10 marks in an area you could use 2 (5 mark) word problems or 5  
(2 mark) sums.

I wanted to see if children given a chance would just do sums and check  
meta-cognition - can the kids really do the questions they come up with.  

One of the first questions the children asked is if they can make the  
questions really simple. I let them know that this demonstrates what they  
can do, so if simple questions is what they can do, so be it. The idea  
of demonstrating how much you can do appealed to them though I was  
not sure to what extent it would be implemented.

I mentioned that I expected them to understand what they were doing and  
not just copy a question from an example and repeat it by rote. I  
mentioned that I would change the numbers in the question paper if I felt  
they were too tailored.

Though I did not allow the cheat sheet that I usually do, stating that its  
their own examinations and they know the questions I did allow it to be  
open book on the examination day.

The children were very excited with the prospect and worked seriously in  
class, through only a few students worked at home to complete it and and  
we made most of the question papers over two classes. The classes also  
served as an opportunity to review some of the topics that the children  
wanted.

Come exam day, Fif (clinical clown) had  
made a bunch of clown noses with  
plastic bottle caps, so it was quite a  
sight with the kids writing their  
examinations with clown noses. One  
kid in her feedback even remarked that  
this doesn't feel like an exam, it feels  
like a party.

I used this time for retrospection and  
did a one on one interview with each  
child. The inputs were interesting and something I need to ponder over.

I have put together a chart of how each of the 17 children organized their
paper in Level 1: sums (blue), Level 2: advanced sums with multiple operations (brown), Level 3: Word problems and those involving abstraction in algebra (green). These fill up the score of the bottom 100. It was interesting that almost all children tried at least 10% of word problems. The graph also shows how they did in their examination from 100-200, 7 children were able to do more than 90% of what they felt they could and 6 of them were confident enough to use more than 40% of Level 2/3 questions in their paper.

5. Adding inquiry to science exhibitions

a) Preparation and discussion before the exhibition

The 6th graders are putting together a science exhibition for the school. A couple of the students had chosen the Cartesian divers as their project and wanted to present it.

Their presentation lasted 20 s. One child pressed a bottle with the vial inserted while the other gave his spheel. "The syringe goes down when you press the bottle, because of the air and water pressure pushes water into the syringe increasing its density and making it sink."

This is quite stark to how we learnt about them and then I realized that this is how children speak at science fairs. [In its context, a fair has a large number of exhibits and short attention spans, everyone is looking for something new and cool and hoping to learn a quick keyword to explain it all (osmosis, multiple reflections, citric acid, dynamo, zinc+copper). Sixth graders came back from a recent fair knowing keywords and assuming that it meant that they knew concepts e.g. we know density because oil floats on water (which incidentally started the Cartesian divers affair...). Tenth graders who have been there and seen that found nothing interesting (new).]
I could only ask - "Was that fun?", to both the kids presenting and those listening.

We started talking about, if it was fun when they were exploring how the diver works. Yes. Ok, what made it fun? We came up with - We had to keep guessing what would happen next and things became clear in time.

We brainstormed on how we can do it differently and include some questions that can be asked of the audience
1) Show the divers
2) Put the bottle in bottoms up and see what happens
3) Ask: What happened? Why?
4) Press without closing cap
5) Close cap and press and watch it sink.
6) Ask: What happens when you press it on top?
7) Ask: Is it water or air pressure?
8) Fill water fully and repeat experiment
9) Explanation (if required).

There were some questions for clarity
1) Can this happen with any other object?
2) Will it work in any bottle?
3) Will it work with 2 syringes?
4) Will it work if the bottle is only half full?
5) If you cut out the heavy side of the syringe will it work?
6) What is the name of this thing?

There were some additional suggestions:
1) Make model with all three bottle settings open, partially filled, full.
2) Both people should speak
3) Talk about how much pressure is required for half and fully filled bottles. They are not the same.

I asked if anyone else thought that their projects simply does not fit into this sort of example and got a few, this led to some wonderful discussions on not limiting ourselves to one example, but looking at phenomenon as a whole and exploring more possibilities.

I felt what was missing in the presentation of the Cartesian was that we were using a readymade set and didn't build from scratch. The next morning on my walk with Shifu (I found a straw on the road), a shampoo bottle cap from the trash and borrowed modeling clay from my son and set out to make some Cartesian divers.

Looking at these divers the kids felt that they could really make it their own and returned my (purchased and self made divers) and committed to make one themselves for their presentation.

In the straw model I put a hole on one of the ends (I remember seeing something like this on instructables). In both cases you add enough clay to just make the object float.
An interesting conversation started in the class regarding the relative speeds of the different divers.

**b) Execution at the exhibition**

The science exhibition came and went more or less as it usually does. I briefly went to the exhibition and it seemed the wave of - "experiment description, demonstration and explanation" had pretty much wiped clean any possibility of getting people to think and getting them to figure things out on their own. Our detailed analysis of how we could present the Cartesian divers was also unused.

To the credit of the children, they did make the divers on their own and even attempted a new kind of diver that we made with a balloon and some weight. However, when it came to presentation, everyone developed cold feet and sprout out the explanation.

At the next class I asked if anyone with exhibits asked any questions at the exhibition. Five hands went up, the questions were:
1) Which of the vessels sounds louder?
2) Why does the vial go down when you press the bottle?
3) Will this float or sink?
4) Why can't you see the coin when we point water into the glass?
5) I asked them to hold the corn seed between the finger and thumb. The first four corresponded to straightforward experiments. The fifth was intriguing because it was not a question, so I inquired further.

5) Pree said she asked people - "Can you hold the corn seed between your index finger and thumb?". Pree happily added, it only popped out for one person, but most others could do it.

Since I still didn’t understand, Pree gave context: *I had made this chart for solid, liquid and gas and put up a table with these in containers. My chart said that solids have definite shape and liquids and gases take the shape of the container. In the last class you said that salt also seemed to take the shape of the container I had put it in, this confused me. I could not change my demonstration chart or my collection of boxes, so I asked people to hold the corn seed between their fingers and they could do it. You see, you can’t hold a liquid between your fingers.*

Ah! Inquiry lives :).
I decided to drop being subtle and brought up a discussion of what the kids feel a science exhibition should be. I asked them to answer the question they could use what they enjoy most at an exhibition:
1) *The presenter should be able to talk about it in both Tamil and English*
2) *There should be games that you can take part in*
3) *There should be something you can turn on and it does something*
4) *It should be unexpected*
An science exhibition with interactive exhibits could be something for us to think about...
6. Integrating electronics and mathematics

The children in the school, even the ones who are not taking electronics classes have been excited about the lab. I would get asked for components or enamel wire by students that I had not worked with. Given the level of excitement I felt that it was tap into it to give an context for mathematics.

As the Electronics lab has become established I have started to take in kids from 6th and 7th grade for an Electronics class in one Math class a week. The kids have been pushing me to include some electronics in their 'curriculum' and when I found one of them in 7th grade trying to make a lemon battery in my class I could not let go of the opportunity. I usually take half the class and let the others continue with the worksheets with the Math teacher.

For the lemon battery they had a small piece of zinc that had been broken out off an old battery casing. They had wound it to a copper wire.

We talked about a new quantity voltage that gives in the water analogy the height of the water it is raised to and measured what we got from the lemon it was around 0.7 V. One kid was consistently able to find out which end is at a higher potential by putting the leads in their mouth. He said that the side he got the shock from was positive. I didn't try it out, but it was accurate every time I asked him.

We sawed through a battery carefully into two disposing off the material inside into a box and then flattening out the zinc plates. We then cut them into strips that we can use.

We talked about an LED and how unlike a light bulb it works only one way. Most kids have salvaged some LEDs from somewhere and are somewhat familiar with them. We started talking about ways (including the use of the multimeter) of figuring out which way it should be connected. We found four. The length of the leads (if available), the cut on the LED, the shape inside the LED and using a multimeter in the continuity setting. I then told them that they needed more than two 2V to light up an LED.

We used both his lemons and were still around 1.3 V still short of lighting the LED, as we had no other lemons I used a 1.5 V C battery in series. It got us to 2.8 V and enough to light the LED and light it did. Then we tried to squeeze out the lemon juice in bottle lids and were able to get enough voltage in four lids to light the LED.

I built on the experience with a few Xth grade kids as they created sets of wires with copper wire soldered on one side and a zinc plate on the other. I used this with the 6th graders. With 4 lemons and 6 kids it was fun exercise. They connected up two and when they hooked up the third the voltage went down. Then they started talking about negative numbers and remembered that the polarity is actually important and its otherwise like adding a negative number. I couldn't have come up with a better example of a negative decimal number than the one they created. I considered making a game of it, but the kids looked too serious to disturb so I let
them at it till they got enough voltage to light the LED and look quite satisfied with themselves.

The other aspect that was easy to address was conversions from milli of a quantity to a whole e.g. mV -&gt; V. This happens quite naturally as they try to interpret the results given from the multimeter based on the range they put it in.

I've also had a few lec-dems using an oscilloscope to explain the differences between AC and DC and walk them through the conversion.

We have also been working on building a torch with a 9 V battery and that an LED gets burnt out if you directly connect it across the battery (we of course, tried it out). To protect the LED we used resistors and started to look at different types of resistors and noticed that the larger the resistance the smaller the current. I plan to revisit this next year with the multiplication and division stories with voltage, current and resistance.
7. Surveys with the children of 6th and 7th grades

a) 7th Grade Survey
At Udavi in the first term I had worked with a few kids from 7th grade in sort of remedial classes using teaching aids. In the second and third term I took the entire class (with the teacher observing). I had two classes (1.5 hrs each) a week in the second term and three a week in the third. This was that I initially relied on class room delivery - use of teaching aids - densiel blocks, geo-boards, pizza party. Slowly as I got more comfortable not sticking to script, we worked on basics stories to understand multiplication and division and extended it to fractions, puzzles, algebra, practical geometry, geogebra (connecting algebra and geometry). We were able to do one project on common mistakes children make in fractions, algebra, decimals using the computer and incorporate some electronics classes for interested students on LEDs, measurements and building lemon batteries.

At the end of the year, I wanted to do a survey with children on aspects that would help me understand how children view themselves and their learning and also take inputs to improve the classes for next year students. I took some inputs from a survey conducted by Sri.com to review usage of Khan academy resources in Math and made it much crisper.

I took the surveys through one-on-one conversations to translate the questions into Tamil, if required, and get individual inputs. I zeroed in on the exam time being the best time to get time to talk to children without them being biased by other children's inputs. One issue was that some of the children were tense or worried about the exam, however, they had made their own examination and I expected this to be lower. The first question, 'how do you feel?', also gave the children an outlet for their emotions. A child who had lost her grandparent a couple of days back was sad and expressing her sadness helped her notice it and feel listened to, after a couple of minutes she felt ready to participate in the survey.

I asked for a couple of things they had learnt well this year and areas like making an abstract equation real with a story, algebra and its link to geometry, fractions and puzzles figured often in the list. Of the 17 children, 12 said that their goal in the next grade was understanding and learning new things, 5 felt that their priority was getting good marks.

During my very first interview I added a question about what a child expected from his/her teacher because this is how the first child interpreted the next question. It seemed something she wanted to talk about, I thought it would be interesting to see what children had in mind. A teacher who is friendly, understanding, provides clarity and introduces something new constructed this image. One reply was, they should be like me. This kid likes to have a good laugh and I looked up from my laptop to see if he was kidding, but his eyes had no humor just earnestness. Ok, sweet.

9 children felt it mattered to them what the teacher thought of them and primarily some of those said they cared about what other students
thought of them.

12 kids said they liked math, 4 others said they like it at times. One child felt that he doesn't enjoy math because he only seems to enjoy it when its extremely easy. Another child felt that he needs to feel challenged to enjoy math.

I'm not sure how children interpreted - do you learn easily? The children who struggle in class can see it and the ones who are doing well know it too, but for the rest I am unable to process any trends.

The replies on grit - can you continue to work when things get tough or boring suggested that most of the children who struggle in the class in Math said it was difficult for them to keep going when it got tough or boring. This is in line with my current understanding of one of the skills that we need to develop in children.

Suggestions on what could be added or changed in class included field trips, more projects and puzzles, more science, games (I primarily did this with 6th graders), more electronics classes. A few children wanted to go beyond peer learning and teach the 6th/5th graders what they learnt. Three children brought up the need for daily/more classes.

The detailed survey is available in the Appendix.

b) 6th Grade Survey
The 6th graders had a lot more freedom and flexibility in the classroom and I tapped this to work with games - short mental board games, pizza games, (dis-)entanglement puzzles. Moving towards stories of multiplication and its equivalent two division stories, integers, fractions, decimals, areas with geoboard, cartesian divers, introduction to algebra and many puzzles. I worked closely with the class teacher Sudhir and we worked a fair bit on classroom environment where children could listen to each other e.g. multiplication stories and convert it to division stories. We were also able to wait out conversations to subside on their own when the children realized that the class was waiting for them.

Pre said that the one thing she learnt well this year was listening. Listening to each other (and peer learning) was an important aspect of our classroom environment, but for Pre to notice this as her best learning for the year felt special. I wonder if I would have emphasized listening as something I work on, if she had not reminded me.

My best classes have been when I have been happy and calm. When some children say - a teacher should teach well and offer no explanation, I wonder if they are trying to refer to what they want us to be, rather than what they want us to do.

A very large number of children seem to care about how the teacher feels about them (and not what their own classmates think of them). I wonder if I utilize this to always help them grow.
Almost all the children I work with enjoy math, I wonder if they always enjoyed Math or our definition of what constitutes Math makes them feel this way.
8. Electronics classes with Xth grade
a) Basics and small projects
I have been taking a session (1-1/2 hrs) of electronics for the Xth grade kids. This is their only non-academic class in the week. We have so far been working on the basics without looking into any 'theory'.

**Multi-meter**
We got familiar with the multi-meter as a continuity meter and went around school looking for things that will 'beep'. It was a chaotic class, the best kind to begin with and some of the girls even removed their gold ornaments to measure them.

We found that most things that we consider metallic are actually coated with paint and need to be scraped to get to the metal and that rust is not conductive. The children also tried out measuring plants, hands, metal, wood, etc.

We put the fear of getting a shock by getting a mild shock from a 9 V battery. True, there were only two volunteers who were willing to put the leads in our mouth. It was agreed that we would not eat the circuits that we build and only work with 9 V circuits for now.

We then switched over to measuring resistance of various objects that have a high resistance, but not insulators like wood. We used water, moist skin, etc.

We also took some 1/4 W resistors and measured them as groups and got a handle of a range of resistors from a few ohms, to k ohms to M ohms. We did a few conversions between kilo and Mega and the children looked through their measurements and found that water had the highest resistance that was still measurable.

We understood how to do the above with a multi-meter that required the range to be fixed manually.

**Bread-Boards**
By now I had got a little more organized and bought a few prototyping boards (bread boards) and we used the knowledge of using the meter as a continuity meter and finding out how the board was hooked up. This took a little longer than expected, but it was perhaps better than explaining it each class. They also built their first circuit of LEDs in series with a resistor and the battery.

I also got notebooks for all students so I can track what they understood (no, they couldn't copy from the board yet as I didn't have a board).

They learnt that putting an LED across a battery causes it to burn out and it does look different once it has burnt out.

I didn't have enough batteries and had to dig out my stock from a long time back and got all kids of different voltages with the 9V battery. One child tried to put a switch in series with his circuit, but was unable to figure out how a two throw two pole switch works.
Two-pole two throw switch
By now I'm a pro and start things exactly where one of the kids gave up and gave them the nasty two pole two throw switch to put in series with two LEDs and a 1k ohm resistor.

We spend 15-20 mins figuring out how the switch works and then the groups put it together. It takes time one group manages to hook up a circuit where both LEDs light up with the switch is off and one lights up when the switch is on (they put the switch across one LED). Fantastic, now they have a circuit to debug and once they do I let everyone know what the group was able to do. What they got one LED to glow in one position and two in the other, everyone is trying to figure out how to do that now. The wheels are turning and some people come up with an alternative way of doing the same, cool.

I ask everyone to draw one of the circuits. I get feedback that they are not artists, but the LEDs looking like light bulbs starts to light up the notebooks. Some draw the exact dimensions of the bread board. I break their art class and introduce symbols for various components. There is still confusion as to how to represent the bread board, I decide to deal with this later.

Some of the folks are finished. I ask them to use the switch to light up one light at a time. They are catching on and starting to use both the poles of the switch though this is confusing their sense of direction of the LEDs. Great.

Serious Abstraction
I felt it was time to take the Electronics class up a notch. I do an assessment. Well, an assessment is not a test, of course you try telling that to children who have heard all the keywords in the world :).

We have been using a two pole two throw switch as a one pole two throw switch (actually as just an on off switch). I thought it was time to introduce the kids to the true power of the switch, series, parallel circuits and also throw in some real life circuit that most people have seen, but don't know how it works.
I split the 16 kids in pairs of two which allows them someone to discuss and debate with, but does not turn into a brawl. The rules of the game are - you can talk to your partner all you want, you collaborate, but you do not cheat i.e. you can get ideas from each other, but you can't just copy what your partner does.

After giving the assessment due respect with a few groans, we get cooking. In a couple of minutes some have oversimplified the problems, but most are in serious discussion. A few of the kids are a little faster and write reasonably long stories to explain what the circuit does. I ask them to keep it brief, but accurate. We come up with conventions LEDs get labels L1, L2 to differentiate them. The switch positions get labels A, B. The answers now look as simple as

SW in A: L1, L2 glows.
SW in B: L1 glows

Nice, we are actually mathematizing the problem.

In time the kids finish 2-3 problems, now, they want to know if their answers are right. I must admit I gave into the temptation and asked some of them to think some more (which of course meant it was incorrect), then I got smart. I told them they could build it and find out. Great, mad rush to get their breadboards (which is nice for an Electronics teacher), except, new rule, you can use the breadboard once you have solved all the questions to check your answer. I am quite impressed that the kids did not take a short cut and continue to work seriously.

It helps me figure out what each child is missing, I fill in some gaps by questioning some of their assumptions. The kids push to figure out if they are doing right I push back with poker face (memo to me: poker face needs more work).

Four groups finish and put the circuits together. The two throw switch gives them some trouble, but they put some of the circuits together and confirm their answer. But, doubt still creeps up and they need an
affirmation from me to see if its ‘right’. They try to get a confirmation, finally decide to figure it out by themselves.

From having only drawn one circuit before and being all confused about nodes (where you measure independent voltages) before the class to having drawn 6 figures (or more), figured out how to label nodes and name components, to making sense of how the circuit would work...my work was done.

**Mini Projects**

With breadboards the children did projects on oscillators using voltage regulator, transistors and then with the timer IC555. They lit LEDs and using a variable resistor even made a variable frequency oscillator. They realized that it could be connected to a speaker and were able to make a sound generator with various notes.

From here they started working on 7-segment displays and counters to be able to count up from 0-9 based off the oscillator.

**Oscilloscope Demonstration**

I borrowed a single probe oscilloscope from the Auroville ITI for a few weeks. There is nothing like an oscilloscope to understand transient signals. I was able to use it with the 10th graders at Udavi who had been working on time changing circuits and with the 5-7th graders in Isai Ambalam (IA) who had been working on understanding Energy and had visited a solar energy company in Auroville and encountered AC/DC, but didn't know what this really meant.

The primary demonstration I gave was of a voltage regulator (AC mains to 12V ’DC’ output) - transformer with center tap, half-wave, full wave rectification and what happens when we add a capacitor followed by what happens when you put a load across the capacitor.

Having only a single probe I could not show them that a center tapped transformer has in and out of phase components at its output. This had to be inferred by the full wave rectification.

I was able to couple it with a cute experiment I discovered of taking capacitors of varying values, charging it by touching them to a 9 V battery and connecting it across an LED to see that the time for which the LED is on increases as the value of the capacitance increases.

The 10th graders made a couple of astute observations at the end of their class:

1) Sun - Yes, Sanjeev this makes sense. I always wondered why on turning off the power in some devices the LED of its charger is still on for some time. I think this must be because the LED works off the output of the charger which has a capacitor. Is that right?

2) Des - If the output with a load always has dips then we can't use this directly as a good DC. Should we create a higher supply and feed it to a chip like 7805 (voltage regulator IC we had used in class) and then use its output? This way as long as we maintain the ripple to be beyond 5V it will give a 5V output.

**b) Different approaches of the Electronics classes**
1) One area: I had started classes by asking everyone to work on one thing on a bread board e.g. getting a seven segment display to work with a counter, using the 555 timer to generate a clock. The classes were somewhat structured with at least 1/3 of the time being white board based. This helped in getting the ground rules of how to read the pin configurations of ICs, LED displays and how to go about connecting them together. It gave a clear focus to the class and helped me give instructions that could be helpful to many students at the same time. It was easy to manage their questions as they were limited. I also used to throw in, one new thing a class, e.g. thermocouple, demo of AC-DC, that kept the interest of doing something new each class. However, with just one thing to work on it was difficult to keep the interest of the entire class and about 1/3 of the class were not always present: either there was an enthu cutlet in their group that did all the work, or it was not engaging enough for a student in a class and the lights were on but nobody was home.

2) Chaos Rules: To break the monotony of highly structured classes I had attempted letting people take anything in the lab and open it up. For about two or three classes the students opened, pried, de-soldered on old TV screen and whatever other equipment I had accumulated over the yrs. The TV bore the brunt of their salvaging and they extracted the enamel wire from its various components with much glee :). There was a marked difference in the confidence of children to build something, knowing that if something went wrong, they could always take it apart. Although, this gave them a sort of undo button, beyond that not much happened. They didn't necessarily want to know more than the name of something the pulled out (as they were not using it). Is this something good to remove? Yes, its a high voltage capacitor, it can take up to 200 V. Great, whatever, lets desolder it. After a couple of these classes I sat the group down and asked them to list what they had learnt. They said they enjoyed it and learnt how to take something apart, but it stopped there.

3) Two projects: As we started working on the Arduino I transitioned to two section classrooms. In hardware I asked them to solder what they had built before a seven segment display to a counter and wire it up so that it had a common interface. I had asked the software group to wire up the display with resistors to the Arduino so they could count inside and display the digits. One of my goals was to demonstrate the difference between hardware and software. With the hardware I wanted to string a bunch of counters and show that you can, in principle, extend it beyond what Arduino is capable of (limited by number of outputs). For the software group, I had planned to count backwards as well and demonstrate flexibility that software provides when used along with hardware.

They learnt to solder with a clear goal. The software group that was wiring just the LED with resistors with the Arduino were able to get their projects to work in a couple of classes. The tricky part for them was to take the wires and order them as a bus and solder them to a connector. Soldering
wires to a 1 space apart connector is a real test of soldering and their work got neater over time.

The hardware group that had to do the wiring realized that it was much more difficult than breadboarding. As they were learning soldering only one of them was able to complete the board completely and they used the solder for too long and the counter chip burned out. I was able to order some general purpose PCBs with horizontal tracks which would make the connections much more easy and one group that had really put their heart into this has taken up and are doing it again with the new boards.

4) Organized Chaos:
After some introspection, I felt that the kids still wait for me to walk them through every step of the way and my time becomes the limiting factor on how much gets done. I felt they had learnt enough to warrant more independent work.

Over the last three classes I broke the class into groups that were clear about what they wanted to do and started chipping at the rest of the students who had not been as involved.

I have fundamentally changed my attitude to allow myself to be less useful and let the students explore more (ok struggle) on their own while giving a general direction or goal.

Here is an example of what they did towards the end of the term:
1) There are a few kids who have taken what they do more seriously and are working with the Arduino to build an instrument to detect the speed of falling objects.
2) Another is building the electronics for a model of a traffic light (R, Y, G) coupled with a timer that counts down the time with 2 or three digits (again with the Arduino).
3) There is a game to take a loop across an electronics maze and one group is working on a buzzer that will trip and keep ringing once a wire touches the maze (till it is disabled).
4) One group is working on soldering a counter in hardware with counters. They were the closest to finishing the soldering assignment but their chip burnt out due to excess heat. They are now learning how to use sockets and put the chip in once they finish soldering.
5) Two groups were working on two solar torches that I have from friends that I fixed, but the batteries had gone into deep discharge that the solar panels could not recharge. They tried to understand the panels, measured the voltages and tried to replace the panels with power sources that would charge the battery. Hopefully, once they charge the batteries we can re-engage the panels and things would work.
6) One student is working on trying to get an inverter to work. He fixed the fuse, tried to fix an LED that was bust, but it bust again. This week he learnt about relays (from group 3) and was able to check that those still work on the inverter.

Given the limited time (1-1/2 hrs a week) that these students have, perhaps, there can only be learning if they are motivated, exploring and thinking on their own. Progress has been slow, but I can feel the progress. Here are some conversations I overheard and had with the students:
1) One student in group 5 telling another - I'm fixing something real, this is one class I really felt I am accomplishing something.

2) **Student**: Sanjeev we were able to do nothing this class. Why can't you spend more time with us?

**Me**: Really. You didn't learn to compile code and write into the Arduino.

**Student**: You didn’t help so I had to figure it out by myself. But, that's not learning.

**Me**: How about what to do with the LDRs?

**Student**: You just gave us the sensors and asked us to figure it out. We were so confused. We just characterized the LDRs with and without light and know that it changes from 3 kOhm to almost an open. We are not sure how to use it, but if we put it in series with a 10kOhm resistor we get from 1.5 V to 4.5 V.

**Me**: Ok, do you think it can be better?

**Students**: We don't know, but maybe if we increase the 10kOhm resistor and lower the lower voltage. This will help the Arduino trigger properly as it needs as close to 0 and as close to 5V as possible. We can't really explain it, can you explain what's going on....conversation proceeds right through snacks break, but they don't care and want to get it right in the next class. I skip snacks too.

I am having a conversation about design choices (granted simple ones) with Xth grade students from a school catering to village children. Feels like progress to me.

c) **Microcontroller programming**

Five children worked on programming a microcontroller for two projects - one to measure the speed of a dropping ball and create a street light (model) with the timer counting down.

Arduino offers a unique way for children to tap into the power of microcontrollers to build projects and needs to be further got into over the years.

d) **Outside support and OLPC salvage**

Along with the AITI there were also other groups of professors who supported us. The lab is now equipped with both a signal generator and an oscilloscope to allow the children in the coming years much more experimentation capability.

A set of old OLPC laptops were salvaged and Java and Arduino installed on them to be able to program the hardware boards making an extremely cheap solution to create high end hardware projects.
9. Logic in English stories

It was interesting to partner with the English teacher at Udavi who was working on creative writing. I took the responsibility of looking at the logical flow of events in a story. The teacher was already looking at grammar and language aspects and wanted the idea of a story flowing to be handled separately. I received the stories each week and read through them and provided individual feedback to each child once a week.

After about four weeks I realized that the children themselves were able to see the gaps in their stories. In a couple of more classes I could find no more gaps to correct.

For a couple of more weeks I continued providing inputs on how we could make the story more interesting - character building, stakes in the issue, ingenuity of solution to be clearly displayed.
10. Resonance of this work beyond Udavi

a) Math research
The research I did at Udavi was complemented with similar research at Isai Ambalam school as well. Much learning from one fed into the other and vice versa. While details are beyond the scope of this document the idea of allowing children to work at various levels within the same class was something learnt at Isai Ambalam as I was working with a mixed grade classroom there (5th to 7th). The science experiments that were integrated into the math classes were also a learning from Isai Ambalam. Isai Ambalam benefited from many of the classroom experiments at Udavi that were used to improve the class there.

b) Electronics laboratory and interaction of children from different backgrounds
The uniqueness of having an electronics laboratory at Udavi brought in interested students from other Auroville schools including few children from Deepanam school and a few from The Learning Community (TLC) to visit Udavi. The classes were enriched by the interactions of the children with the 6th graders at Udavi.

c) Online documentation and outreach
The work at Udavi (and Isai Ambalam) have been documented online at http://smallisbeautiful.blogspot.com/
Over the course of the year over 60 articles have been published on the blog and over 6500 visitors have read these articles. We believe that our small work can have a larger outreach effect in inspiring others to act.
11. Building on work done this year
The developments this year give a clear indication that we are in the right
direction in making math and science more relevant and using it as a
vehicle towards critical thinking.

Future work will include integration of electronics within the curriculum for
6th and 7th graders, more hands on experiments and projects, working
towards a science exhibition that focuses on inquiry for the visitors.

Based on the documentation online, we have received a grant from Prof.
Shree Nair (CS, Columbia University) to experiment with
bigshotcamera.com a DIY high end digital camera. We hope that we can
design and create a program that introduces curricular topics for 6th
onwards by showing its application with the camera.

We look towards this exciting opportunity in the next year.
Appendix

1. Links to detailed Surveys of 6th and 7th graders

*6th Graders Surveys*
*7th Graders Surveys:*

2. Links to photographs

*6th Grade*
*7th Graders taking their examination*
*10th Graders in an electronics class.

3. Teachers notes on specific areas of interest/innovation

**a) Fractions**
Many kids in grade 7 are unable to operate with fractions. Among them a big segment are unable to grasp what fractions are, a second smaller set are unable to proceed on the arithmetic even after they understand why they are supposed to factorize, take LCM, etc.

As I took supplementary classes for the 7th grade kids, it was obvious that they were comfortable operating fractions when the denominators were the same (5th grade). Of course this just meant that they had a system where they added/subtracted numerators without worrying themselves about what fractions are/were and this was a gotcha in 6th. I am also working with the 6th graders to address the issue here itself.

Numbers are abstractions, but fractions are more so given that they are parts of a whole and a fraction can take different avatars depending on what the whole is. E.g. 1/2 of 1 kg is 1/2 kg, but 1/2 of 1/2 kg is 1/4 kg. A nice abstraction of a whole is something circular. It makes it very obvious when pieces cut diagonally are extra or are missing.

**Pizza Party**

Most children around Auroville actually know what a pizza is (not all like it) and I spent quite some time with a teaching aid called pizza party (Creatives). The kids that don't like pizzas assume that its a dosa. The game is fairly cheap (Rs.165) and is generally well done (though the suggested games need work).
What it has:
Base cards of fractions 1/2, 1/3, 1/4, 1/6, 1/8. Pieces of the same proportions. A die that has these fractions (5) and creative written on it.

Modified/invented games:
1) Getting familiar with the pieces using base cards. The idea was to roll the die and pick up the corresponding base card. Then in turns roll the die till the fractional piece on your base card comes to complete the pizza. Well the kid getting 1/2 needs 2 pieces and the one with 1/8 needs 8 so the game if far from fair.
I tried to even out the odds by allowing them to check if the piece fits into the pie and taking as many pieces as it fits, so a kid with 1/8 base card can take 4 pieces of 1/8 when he/she gets 1/2. Now, it seems 1/8 has the advantage. But, we added a no overflow rule i.e. if you get a 1/2 and a 1/8, then you get another 1/2 i.e. you can't use it. The only disadvantage is for 1/3 base card who really do need to wait for 3 such cards to complete the pizza.
2) Complete the rest of the pizza: Roll the die make the rest of the pizza with the pieces you have.
3) Selecting pieces to make a pizza in turns, but you pick the piece for the next person to play. The one who completes a pizza gets a point. You can keep the full pizza as part of the game to see when it gets used and who gives it to who :). Nice game to get into the kids psyche.
4) Pizza delivery game: Group game, the next piece of the pizza is determined by the die roll and the team tries to build 5 pizzas for delivery.

Initially, the children who were getting it wanted to play the game with less luck, but given the mix of kids they ended up playing many different ones. Some kids also wanted new games and we introduced ones with subtraction of two fractions and finding pieces that match the difference, or a piece that is just larger or just smaller than the difference.

One often wonders if doing these fraction games is really worth the time and I started a conversation with the kids of what we learnt (not what we did) from the activity. With feedback starting from 4 pieces of 1/4 makes a whole. This helps reiterate 1/4 means you cut the pizza in 4 pieces and take one. Similar ideas continue for 2 pieces of 1/2 and 8 pieces of 1/8. Then we move on to expressing one set of pieces in terms of another, 1/4+1/4=1/2, 3x1/6=1/3 and my personal favorite 1/2+1/3+1/6 is a whole pizza. Wow, made my day.

A note of caution for teachers using mixed fruit to teach fractions. We ask children to treat everything as 'fruit', adding apples to oranges to make up a whole. The whole is not obvious as a fruit can be added or removed and it would still be a collection of fruits. We then ask them to remember the 'fruits' individuality by asking what fraction of the fruits are bananas. The children get
comfortable adding grapes to watermelons, but they will also add 1/2 a pizza slice with 1/8 pizza slice to give 2 pizza slices.

Equivalent fractions
An often skipped section to work quickly towards factorization and LCM is the idea that a fraction can be expressed as equivalent fractions.
By now, most kids can tell stories about fractions.
What is the story of 1/4? You take a pizza and cut it into 4 pieces and take one piece.
The idea can be extended into the realm of 1 out of every 4 pieces. This helps build equivalent fractions. What if you had 8 pieces in the pizza then 1/4 would cover 2 pieces (from the pizza game). So $1/4=2/8=3/12=4/16=...$

At this point you can reintroduce the idea of adding fractions with the same denominator say $1/8+3/8 = (1+3)/8 = 4/8$
and remind them that the denominator indicates the number of pieces you cut the pizza into. You can add the number of pieces as they are the same size.

1/2+1/4 the pieces are not the same size and can't be added directly. This can easily be seen from the pizza game. With equivalent fractions we can talk about what 1/2 will be if the pizza is cut into 4 pieces. One out of every two gives 2/4 pieces. Now adding: $2/4+1/4 = (2+1)/4 = 3/4$

Most smaller fractions can be added by writing them in equivalent fractions and looking for a size that is common to both.
$1/6+1/8$
$1/6=2/12=3/18=4/24$
$1/8=2/16=3/24=4/32$
This gives $1/6+1/8 = 4/24+3/24 = (4+3)/24 = 7/24$

I introduce LCM after I ask them to add
1/2+1/200 at which point most children start taking a short cut into 100/200+1/200.

Of course nothing works for every child, but I was able to address 90% of the children’s understanding this way. Of course, mastery requires further rigor.

b) Negative numbers, number line
Motivation of using negative numbers is done nicely in picking a set of 10 numbers where the large numbers add and the small ones subtract and trying out various orders of doing this to convince yourself that the result is always the same.

Then if you go around and change one of the numbers by say 1 we know the result and do not need to do all the calculations all over again. However, we loose this magical property when one of the steps require you to remove a large number from a small number and we say that the result is zero. This wall of zero can be crossed for these magical rules to continue to hold.
A ‘kadan’ (loan) can be used to talk about how we are not just inventing these numbers, but we have encountered this tracking in real life. I had an odd confusion from children at this stage. One child could not understand the idea of loan as a negative number. The child’s story was if I borrow Rs.5 from my friend, I will have Rs.5 and owe him Rs.5 so I have Rs.0 not negative money. This story bypasses the need for such a loan and first clarity on something children understand like I had Rs.10 and went to buy a chocolate. The chocolate cost Rs.15. I told the shopkeeper that I would give him money later and took and ate the chocolate. Now I have a loan of Rs.5.

However, limits to these stories are quickly encountered as children are asked quite early to do 5 - (-4) which baffles them quite a bit.

We looked at a number as representing jumps positive number representing jumps towards the right and negative numbers representing jumps to the left. The process is slow and tedious. But, we made puzzles with these trying to guess the order of numbers from the start to finish and worked through it. However, it did not seem to jell well with what they already knew were positive numbers and the jump and the position were clashing. Of course a double negative and any story I tried to convey of the same only backfired.

To add speed I introduced the concept of a vector. A vector is an arrow with a direction. A number is the length of the vector. The vector points right and every negative sign requires it to point in the other direction. The tail of the first vector is placed at zero and all other vectors are placed with their tail to the head of the previous vector. The result is evaluated by reading out the value at the head of the last vector. We first tried positive numbers to see that this works like a charm. Then we worked with positive and negative numbers without crossing zero and then using it without any restrictions. Strangely the idea of introducing a slightly more concept seemed to create a new consistent space worked for them. They soon felt that they had got it and could just work with problems without needing to draw them.

c) Algebra
Early conversations with Xth graders
We talked about an algebra equation as a 'mystery' story. We started small, most children were comfortable with short stories I have 11 books, I bought some more and now I have 19, how many books did I buy?
11 + x = 19.
I mixed it up a bit and we built up to: A man has three sons, to his eldest he gave half his cows. Of the rest he donated two and split the remaining between his other two sons. If the youngest got 4 cows, how many cows did he originally have:
(x/2-2)/2 = 4

The focus was on getting stories and not getting lost in trying to solve them. It got interesting with equation with a second power :) and then higher powers.
One interesting thing that happened in class was when one kid while solving linear equations looked back and asked me if there was a story for that too.

I used a familiar puzzle...
Two brothers (Ram - elder and Shyam) were carrying sticks. Shyam says, brother if you gave me one of your sticks we would both be carrying the same number of sticks. Ram smiles and says if you give me one of yours I will be carrying twice the number of sticks that you carry.

I was connecting the dots in class with what they do in algebra of solving linear equations is equivalent of solving for the intersection of two straight lines on a plane.

It's been fun and I have received some interesting feedback - some kids think its great that they are starting to understand that what they study has some use in the world. At the same time a small minority were happy just substituting values in formula and are getting all confused when asked to look at it graphically ;). Ah well...

**Introduction of what algebra can do with younger children**
I have been doing this through puzzles that can be logically thought through to come up with a solution (which in many cases ties in one-to-one with the algebraic expression). For each new problem kind of problem a new way of solving would need to be thought of, till ultimately algebra can make it all easy (if you can write out the problem)... I had good success in motivating the 6th graders at Udavi that they absolutely demanded to know what this algebra business is all about. When introduced they were even able to write out the expressions though they got stuck not being able to solve it.

**Variation 1:** 120 biscuits are divided between 16 pets (cats and dogs). The dogs eat 9 biscuits each and the cats eat 7 biscuits each. All the biscuits are used up. How many dogs and cats are there?
Approach without algebra: Since each pet eats at least 7 biscuits. We given 16x7 = 112. Each dog now needs two more biscuits. The remaining 8 biscuits are eaten by 4 dogs. The rest 12 are cats.

**Variation 2:** 110 biscuits are to be divided between cats and dogs. The dogs eat 9 biscuits each and the cats eat 7 biscuits each. There are two more cats than dogs. How many cats and how many dogs?
Approach without algebra: Give the two extra cats their 7x2=14 biscuits. The remaining 96 are divided into equal pairs of cats and dogs. Each pair of cat & dog eats 9+7=16 biscuits. There are 96/16 = 6 such pairs.
6 dogs and 8 cats.
The kids got the hang of the variation that we discussed in class and were even able to make their own puzzles and solve it changing the numbers.

**Back to Basics**

I realized that the best way to address algebra was going back to basics. I started with addition and its two equivalent subtraction stories.

I had 5 apples, my mom gave 3 apples. How many apples do I have now?

Becomes:

I had some apples, my mom gave me 3 apples and now I have 8 apples.

I has 5 apples, my mom gave me some apples and now I have 8 apples.

Once children were comfortable with this story, it was appropriate to introduce that we can put a quantity we do not know as x and rewrite the above stories as

\[ x + 3 = 8 \]

and \[ 5 + x = 8 \]

the children were then able to relate to the fact that even though we are saying something plus 3 is 8, it is actually a subtraction story and are able to evaluate x.

We then moved to the multiplication and division stories to understand why the reverse of the operation works when dealing with algebraic expressions.

d) Using Geogebra (intuition in two variable algebra)

1 : Geogebra : introduction

We had worked on one variable algebra with puzzles and looking at abstracting something real and taking an abstract equation and interpreting it in real terms. I introduced geogebra as part of this effort to get an intuitive feel of what two variable algebraic equations look like.

On the resource front computers were available for all the students of 7th grade a couple of days a week which could be used for some unique learning.

To get comfortable with the tool I worked through practical geometry. I had worked with one grade for 2-1/2 weeks to construct 90, 60, 30 degree angles. After a demo (only geometry) in one class their first assignment was to draw an equilateral triangle for a line segment they drew, measure the sides (to see if they are equal), measure the angles (to see if they are equal) and label the vertices.

I noticed how quickly the children become comfortable with a tool that is marginally intuitive and how differences like lines and line-segments become quite clear as they try out different menu
options. The references of constructions become obvious e.g. for a circle what was the center and which point was used as reference for radius. The measurements add a self-check so the children notice something is incorrect and try to figure out what could have gone wrong. Perhaps, if I had introduced practical geometry this way I would have saved a lot of paper from having holes and trying to make sure everyone had compass, scale, protractor, etc.

Not everyone was interested in trying out geogebra and in the first class I focused on the early adopters and allowed the rest to work on mathlab (computer games that are based on math concepts). By the second class everyone was in and some of the students who had been able to complete their assignments well were paired to support the ones who were doing it for the first time.

For those who had completed I asked them to make a square with each side 4 cm using arcs and lines. It was a fun challenge and one child claimed that he did it. Most others were able to get squares, but not of the same length as their original line segment.

2: Story for x+y=10
I split each class that we use computers into a primer and an execution. In the primer I asked them for a story of x+y=10. I got some interesting replies and I wrote them down and we analyzed them together:
1) I have 2 mangoes. My mother gave me 8 mangoes. How many total mangoes do I have?
2) My sister has 5 sweets, I have 5 sweets. Together we have 10 sweets.
3) I have some mangoes. My mother gave me the same mangoes. Total mangoes are 10. How many mangoes did my mother give me?
4) I have some oranges, my sister has 10 more mangoes than me.
5) Total biscuits are 10. The dog ate 6 biscuits and the cat eats something. How many biscuits does the cat eat.
6) Ramu and Ravi collect some sticks. The total number of sticks is 10. Ravi collected 2 more sticks than Ramu.

There was generally a fixation with having one 'answer' which came from the one variable one equation situation they had encountered thus far...

I started listing the specific examples they had given 1) and 2)
\[ x + y = 10 \]
5 + 5 = 10 (possible, but only one of the possibilities)
2 + 8 = 10 (possible, but only one of the possibilities)
this was enough to get them kick started in getting a load of other possibilities
1 + 9 = 10
3 + 7 = 10
4 + 6 = 10
0 + 10 = 10
and their flips.
I mentioned that there isn't just one answer like they have always been used to there are simply many possibilities.
I also pointed out that if one value increases then the other decreases as the total is the same. We took an example, if Var received 10 chocolates on Sha's b'day and she shared it with her sister, if she decided to be a generous elder sister and gave them all to her sister, her sister would have 10 and she none, if she kept one her sister would get only 9 and so on. They seemed to think I went to too much effort to state the obvious.

We then considered each case of the other stories they had come up with and went through what they mean and if they sufficiently cover \[ x+y=10 \] or not.
3) Is like having twice the number of the mangoes indicating 5+5=10. Doesn't seem to cover everything.
4) You can't add mangoes to oranges unless you start treating them as fruit and forgetting that they are mangoes and oranges. The source of confusion was that \( x \) and \( y \) needed to be different as mangoes and oranges are! But, if they needed to add up to 10 they needed to be of the same kind. The child came back with, ok, they just need to be different numbers not different fruit. Another child corrected its possible they are different and its possible that they are the same!

By now they figured out that 5) was incomplete as I didn't give the number 6. This led us to 6) and that the first part made sense, but where did the second part of the story came from? Arc pointed out that that's how some puzzles were and more condition was needed for an answer. Hmm...I asked them to work with what we have right now and that the reason for the other condition was coming soon.
Back to the possibilities, I drew out the x and y axis and started putting in values for x and y as ordered pairs on the axis starting with (10,0) and (0,10). I put a few more points and then jumped the gun and stated that it turns out that all these points actually lie on just one line. Magic! The kids were impressed and pointed out that it seems believable as every time Var gets a chocolate her sister looses one so the line going down makes sense.

We then handled x-y=10. The kids had of course gotten the hang of it and made the stories. We tried to look at how we could make the stories believable! The best we could come up with were - The fruit vendor delivered some bananas and my sister ate some in the morning and left, now I see that there are 10 bananas left.

We then analyzed that how if the sister took more bananas then there should have been more to start with and how this line (they already guessed that this would be a line) would go up.

Computer assignment, plot
\[ x+y=10 \]
\[ x+y=20 \]
\[ x+y=30 \]
\[ x-y=10 \]
\[ x-y=20 \]
\[ x-y=30 \]

The kids had gotten comfortable with a story of \[ x+y=\text{constant} \] and what it looks like.

3: Story of \( ax+by=10 \)
It was time to move to the story for:
\[ x + 2y = 10 \]
Once we cleared the confusion with
\[ x + 2 = 10 \]
and remembered that \( 2y = 2 \cdot y = y + y \)
The story that Sub came up with was
I have some oranges, my mother gave me some oranges and my father gave me the same number of oranges as my mother. Now I have 10 oranges.

How would this extend to
\[ x + 5y = 10 \]
Now the extended family was joining in and the kids felt that the stories were making less and less sense and I should now start contributing to the discussion.

I reminded them of a simple multiplication story which Arc used to come up with the following
I don't know the price of a liter of milk or the price of 1 liter of oil, but the cost of one liter of milk and 5 liter of oil is 10. Though we all agreed that given the price of milk and oil the story was difficult to believe it made sense in principle. As in case of addition of variables if one went up the other went down, we talked about the relative steepness of the curves without entirely venturing into slope and that we should try different curves in geogebra to get a handle of what happens if these numbers change.

Using geogebra we concluded that what mattered is the relative index of \( x \) vs \( y \). The larger the index of \( x \) vs \( y \) the flatter the curve.

We then used the method of putting \( x \) zero to get \( y \) (y intercept) and putting \( y \) zero to get \( x \) (x intercept) to draw a few lines. Once we drew a line we used the intuition from the stories to check if the line made sense. This helped catch some mistakes when trying to find \( x \) and \( y \) intercepts.

We then pushed into freeing up the limits of what indices and constants were including
\[ 3x - 5y = -10 \]
that involves division of two negative numbers and if something went wrong with the intercepts looked at the final direction of the line constructed.

e) Geometry (Xth grade)
As I went through what they had for Geometry I realized that although they had done coordinate geometry for a couple of yrs, trigonometry for a couple of yrs, they were now going to encounter the idea that triangles can be similar and their sides would hold ratio between them! I also noted that there was no proof given in the text for the same :). Great, I can start with one of the most powerful concepts in geometry that allows me to link with
everything they kinda, sorta know of and I don't even need to prove it.

2) The kids may remember the concepts but not the corresponding names e.g. adjacent angles, corresponding angles. They would, of course, remember vertically opposite angles, but who doesn't. This gives rise to the issue that we can't talk the same language.

3) The kids also know me to generally have fun with puzzles and games and I need to have some connection with writing examinations.

Class 1
I started with asking kids to make a cheat sheet (bit paper as the kids call it) for Geometry by putting in all that they know about it. The kids were excited by the prospect though I made it clear I was not encouraging them to take it to examination.

We started with a small list which was enough for our purposes and decided to add more as we went along:

![Adjacent Angles](image1)
![Corresponding Angles](image2)
![Vertically Opposite Angles](image3)

![Isosceles Triangle](image4)

I added one new one of similar triangles, with just one rule (AA) if two angles of two triangles are equal the triangles are similar. In similar triangles there is a constant ratio between any two corresponding sides.
I mentioned to them that the similarity of triangles is the most powerful of the geometrical theorems and now they can rule the world. We started with trying to prove the 'mid-point formula'.

If one can understand the picture the steps are simple, but I went through many iterations of building up this picture one piece at a time. The biggest bottleneck was that they didn't associate the coordinate in coordinate geometry with a real distance. In a point \((x_1,y_1)\) they were unclear as to what \(x_1\) really was. It had just been a number they needed to substitute in an equation. Well, this was precisely what I wanted to address and I didn't mind going over it again and again. Once that is understood it was actually quite easy for them to take up the new concept of \(\Delta CBD \sim \Delta CAE\) and were even able to supply be the reasons (adjacent angle, right angle). As \(CA/CB=2\) (mid point)

\[
\frac{(x_2-x_1)}{(x_2-x)} = 2
\]

\[
=> x_2 - x_1 = 2x_2 - 2x
\]

\[
=> x_2 - x_1 - 2x_2 = -2x
\]

\[
=> x_2 + x_1 = 2x \text{ or } x = (x_1 + x_2)/2
\]

I proved it for \(x\) and I asked them to prove it for \(y\) at home. I completed the picture as below.
I asked them to complete a couple of problem sets in the book and using similar triangles prove Thales Theorem as well.

I worked out a couple of questions from two exercises and asked them to do the rest by themselves.

Class 2
I didn’t expect everyone to do everything, but I expected someone to do something, and they did.

Four kids (in a class of 16) tried and succeeded in proving the mid point theorem, few others attempted the exercises in the book, as they now looked trivial. Few read the book by themselves for the first time to see what’s really there in it :). However, there was still half the class that was waiting for the dust to settle and for me to solve the questions on the board.(ha)

I proved Thales theorem using similar triangles and asked them to add this to their cheat sheet as well.
I asked them to continue to solve the problems in their book. It didn't matter that they had done half the exercise or looked sheepish that they hadn't started. They all had to work and work independently.

It's surprising how noisy the classroom usually is, it's not the kind of organized chaos the teachers permit its the kind that comes from boredom of kids who are able to solve faster and waiting time of those who don't follow for the answer to be written on the board.

They could not talk or borrow pencils/pens/erasers/scales or any of the other million things they seem to always need in the classroom. The instruction was, use what you have, do what you can and find out for yourself what you are capable of. It was not easy, they had not done this except in examinations. I needed a more supportive setting than an exam and I gave occasional individual attention, at times needed to remind a couple of kids about what we are doing, but we were able to get some peace and quiet in the class (another first). I went around to see how kids were doing and for once compromised and let kids know when they had got the answers. Almost 4-5 kids other than the ones who had already done some work were very kicked and would say, is that all there is to it.

This class was a very big achievement, kids worked and to their ability succeeded in solving some problems and started believing in their ability to do so. The most questions solved were close to 10, but even the least were 3. Usually 4-5 questions are covered in class when the teacher solves it on the board, not bad.

**Class 3**

It was much easier this time for the kids to settle and work this time. I addressed a couple of common issues encountered in the previous class. We talked about the common strategy in the questions to have multiple triangles and a common side that was divide in the same ratio as the others. We addressed ideas for cute questions like ratio of sides of a trapezium that needed an additional line to be drawn to see the triangles.

We also compared altitudes of similar triangles and proved that they are the same ratio (of course, using the idea of similar triangles). This brought us to right angled triangles and all the physics they had done of comparison of shadows of objects and their heights. We talked about how these triangles are the same and what would happen if the time or place of one measurement is different from another and what the assumption is to say that the angle of incidence is the same. We concluded that the rays of the sun need to be parallel between the two objects, times and places of the measurement. Our misconceptions are buried deep, so for yucks I drew the sun, as we drew as kids with rays coming out in all directions and pointed out that this in such a picture the rays don't
seem parallel and any bright objects gives light in all directions :). It was a fun, after a couple of intense classes this whole exchange seemed just what we needed.

We then cover altitudes of right angled triangles that create smaller right angled triangles that (needs rotation carefully) are similar to the original triangle, which further can have altitudes giving similar and smaller right angled triangles.

![Diagram of right triangles and altitudes]

This was a fun exercise and we really needed to be sharp about our rotations. We finally figured out the algorithm to get things right. Start with the corner where the angle is unchanged, mention the corner with the right angle next and finally the corner left out.

**Chords tangents**
The next topic within geometry was on properties of chords, tangents with circles. For a transition from triangles to circles and I tried to connect up the two by showing them that the x coordinate of a cow going in a circle at constant speed is a sine wave :). This was way beyond the syllabus and had the teacher browse through the textbook to see where this 'stuff' was. It was a quick transition that touched on trigonometry and gave me a chance to plant a seed that I would water in my electronics class with them as I introduce AC signals...

In my preparation I realized that they were supposed to know some really neat stuff with chords and circles in 9th grade. An important one being given a chord any triangle it makes with any point on the circle (on one side of the chord) has the same angle. I tried proving it myself, I couldn't. I didn't find the 6th grade textbook either and finally went though the proof on khanacademy. It had a proof that the angle (subtended by the arc) with the center is double of the angle at any point. Nice, I didn't remember this.
In class I mentioned the theorem and the kids and went a step further and said that the angle with the center is double of the angle it makes, they said that they didn't know it. It wasn't enough that they knew now they wanted it proved :). I could not believe these are the same kids who would take a formula without question and were afraid of proofs 5 classes back.

It's a cute proof that involves a specific case of a triangle made with the diameter to make an isosceles triangle and then generalizes it by splitting any angle as a sum (or difference) of two angles involving the diameter.

I proved the diameter and then gave an example of splitting a random angle as a sum of two angles. They were not convinced that this was general enough and came up with a point where the angles could not be added (point A - a limiting case of summation). I gave them the proof by subtracting the angles. The wheels in their heads were really whirling now. Then found a point on the other side of the chord (point B) and I told them that it was indeed not valid on this side of the chord and we would talk about it later.

There are only a few proofs that are going to come in the X class exam. The advantage was that I could still prove everything but didn't have to write it out in full on the board. We looked at the figure and I made my case by walking them through the thought process while marking on the figure. It's a real class and there are kids that zone out.
The good part is that since I didn't need to write things out it doesn't take time to repeat the proof from where they didn't follow it. There may still be one child with a glazed look and when you ask him/her what he/she missed would come back with ‘didn't understand anything’. In such cases you just start at the beginning, there is a circle and a chord. They come back with Dah, yes I know that, and then you build on it. I have done a proof at max 3 times orally before glazed looks are replaced by knowing ones.

I also needed the opposite angles of a cyclic quadrilateral add to 180 which I again mentioned. By now they were trying to get all that they didn't in 9th grade and asked me to prove it. I drew the angles required for proof and told them they could do it themselves.

We found similar triangles in the intersecting chord theorems (it would have been nice if they had asked them to prove it in the book). Here the two triangles APD and BPC are similar and their sides are proportional.

The atmosphere in the class is tremendously different from what we started in the beginning of the sessions. There is a lot of questioning regarding principles. The children are engaged and there is very little idle chatter. When it is there. I pause the class for the children to finish and I have reaffirmed that its not because they are disturbing the class, its because the rest of us don't want them to be left out. The children also listen when another child is talking and don't try to cut them off.

We are done with the 'theory', now to focus on application of the same.

f) Prime Square minus one
I had a few interesting classes with 6th and 7th graders around the idea that a prime number (beyond 3) square minus one is divisible by 24.

The biggest difficulty in approaching this topic is that many children get confused between double and squares. I indicated the difference using an area of a square of a side of a certain length (avoiding 2 so as not to add to the confusion) vs a rectangle of the same length and breath fixed as 2. This, however, was not enough for all children and I introduced a short cut to get squares of 2 digit numbers. Even
though this was a diversion, it reiterated the squares of small numbers and helped them see squares of large numbers not just as random numbers.

The method is
1) To write the two digit number say 24 in two columns. Now you have single digit numbers in each column. 2) Take the units place which is 4 and get 4x4=16 and write it as a two digit number in the right side of the column.
3) Then you take the tens place 2 and square it to get 2x2 = 4 and write it on the other side of the column.
4) Then you take both the digits 2 and 4 multiply it to get 2x4 =8 and then double it to get 16 and write it in the bottom with the units digit in the first column.
5) Add the two rows you created in steps 2) though 4) as a regular addition.

```
  2 4
  4 16
  1 6
  5 76
```

One trick in the method is when the units place is 0,1,2,3 when the squares are also digit numbers. Here, you need to continue writing them as a two digit numbers - 0x0=00, 1x1=01 2x2=04 and 3^2 = 3x3 = 09. For example 63^2. 2) 3x3=09, 3)6x6=36, 4)6x3=18 double 36.

```
  6 3
  36 09
  3 6
  39 69
```

In the first class I tried this with the children had asked why this was. Being a rookie teacher I thought they meant they wanted to know why you write single digit squares as two digits and tried to explain that the '2' is 20 and its square is 400. Luckily, I also listen to the kids and realized that they only meant that I should repeat this step and give examples for them to master it. (Though it gave me an idea to build on the real why later).

The teacher had displayed prime number all around the class and we picked up prime numbers and squared them (subtracted one) and checked if this was divisible by 24. This is, of course, easier said than it was done. I tried to explain that the factors of 24 were 8 and 3 and we need to do a divisibility test of these two numbers and all these ideas fell flat on their but. Finally, we just wrote the 24 tables and did long division. The issue the children had with the divisibility test is that it doesn't tell you exactly by how much 24 divides the square and till they have this number the division is not real!
This itself was fun and helped as a way of learning squares and gave a practice of long division. What was more fun was to answer two questions:
1) Why does the squaring method work
2) Why is a prime number square minus one \((p^2-1)\) divisible by 24.
I posed the two questions to the children and asked them which one they really wanted to know. If you are an engineer and not a teacher you would be surprised to know that the children only had interest in 1) and 2) was well, a nice side dish.

**i) Why does the squaring method work**
The sixth graders I work with are (have become) a curious lot.

Following the exercise of \(p^2-1\) (prime number square is divisible by 24) there were many questions of how things worked. My goal had been for them to look at the beauty of numbers and I had asked them which their burning question was why \(p^2-1\) is divisible by 24 or how the technique of squaring works.

I had found the property of primes more interesting and as if to remind me that I am an adult the children overwhelmingly selected the squaring method deserved their attention.

There were three aspects that puzzled the kids about the method:
1) Why write the square of the units place is written as a two digit number even if the result is a single digit i.e. \(1^2=01\), \(2^2=04\) and \(3^2=09\).
2) Why the square of the tens place didn't need the same.
3) Why after multiplying the two numbers we needed to double it.
Of course, there was the addition of all these numbers, but apparently that was no puzzle :).

**Distributive property of multiplication over addition**
I needed to explain the distributive property of multiplication over addition to proceed. I realized that the algebraic proof that came naturally to me was irrelevant as they had not encountered algebra meaningfully. I used that place value to show the same graphically.

A place value kit has blocks of ones, tens, hundreds (and thousands that I have now shown).
I had used the blocks to explain multiplication as the area of a rectangle i.e. 6x13 is the same as calculating the area of a shape having 6 rows and 13 columns (or 6 columns and 13 rows depending on how you look at it):

In that case, 6x(10+3) implies a split in the figure above of 13 into 10 + 3 i.e.

Which is the same as: 6x10 + 6x3

As a note, many aspects of algebra can be beautifully shown with the place value kit. I have been able to touch upon the following multiplication, squaring, decimals, fractions, area, volume with the same.

**ii. How do you show that \( p^2 - 1 = (p+1) \times (p-1) \)**

During preparation for my class with the idea that a \( p^2 - 1 \) (where \( p \) is prime > 3) is divisible by 24 I was trying to find a way to explain \( p^2 - 1 = (p+1) \times (p-1) \).

I found the following way with the place value kit. Say you have 13^2. You have 13 rows of 13 each. When you remove one. You get 13 rows of 12 and one additional column of 12. Moving the column to a row you get 14 rows of 12. i.e. 13^2 - 1 = 12x14. It can be seen from the picture that this would always hold.
In fact this is a nice way to see that any $a^2-b^2$ (where $b$
I didn’t get a chance to use it in class, but i thought it was cute anyway.